Nonlinear Filtering with Optimal Transport

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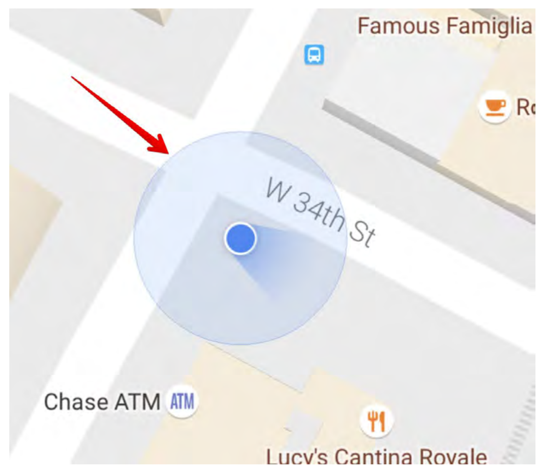
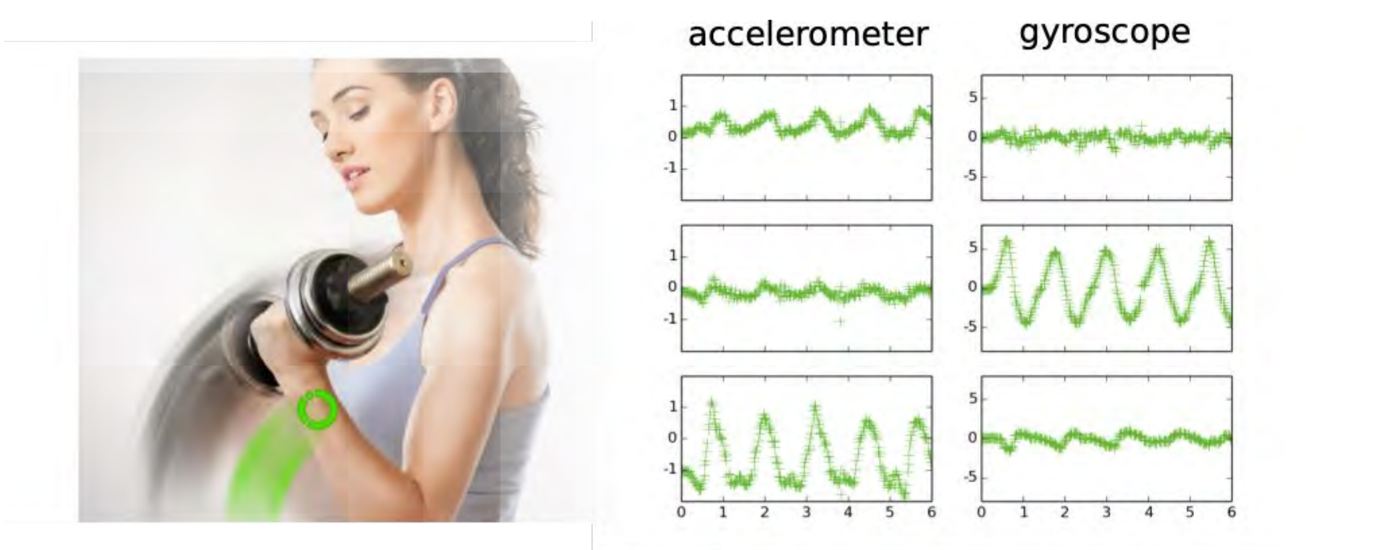
Embracing Uncertainty in Control Systems



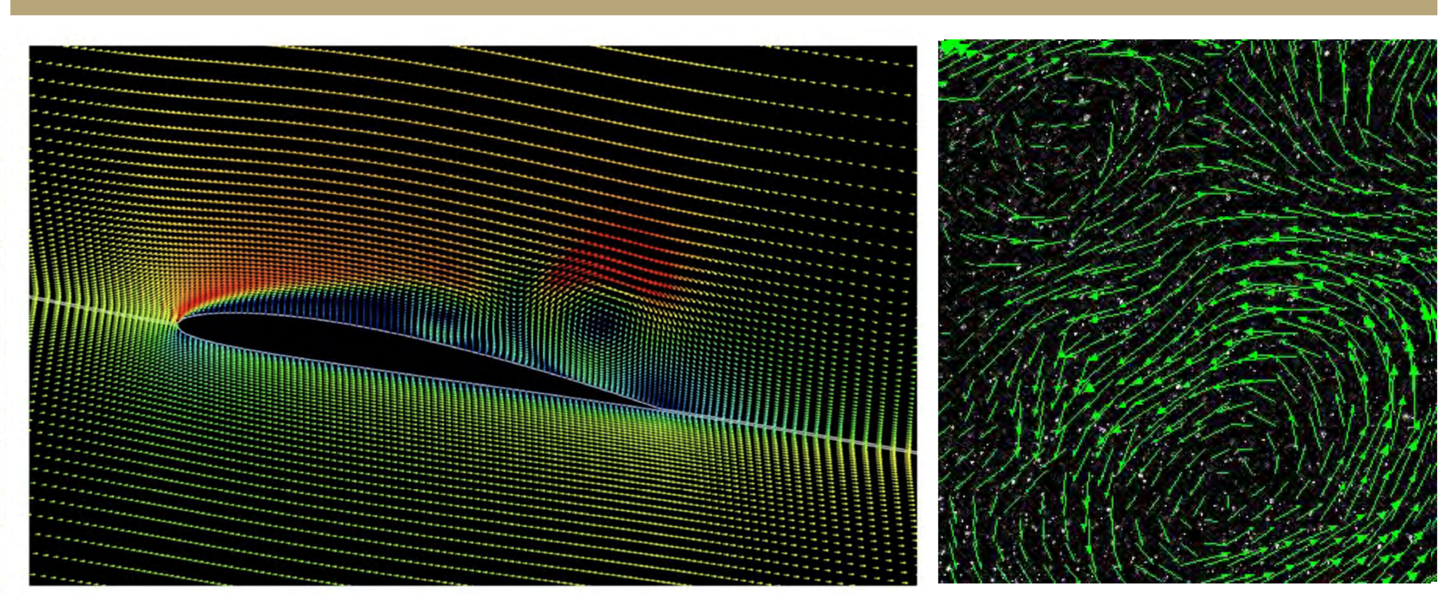
Gesture recognition



State X_t Observation Y_t

- **Hidden state:** Motion of hand
- **Measurements:** Motion sensors, accelerometer, and gyroscope
- **Problem:** Detection of gestures in real time

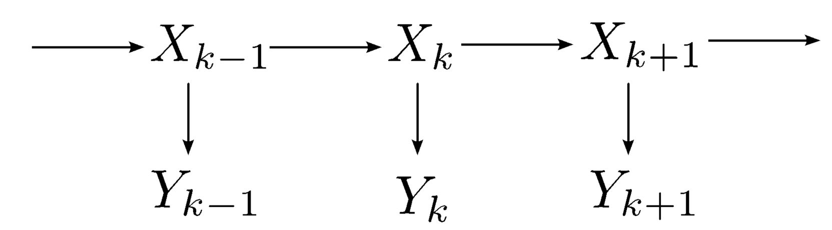
Particle image velocimetry



- **Hidden state:** Particle position/ velocity field
- **Measurements:** 2-D slice image of particle tracers
- **Problem:** Estimate velocity and trajectory of moving particle

Nonlinear Filtering, Bayesian inference

- State process: $X_k \sim a(.|X_{k-1})$,
- Observation process: $Y_k \sim h(.|X_k)$

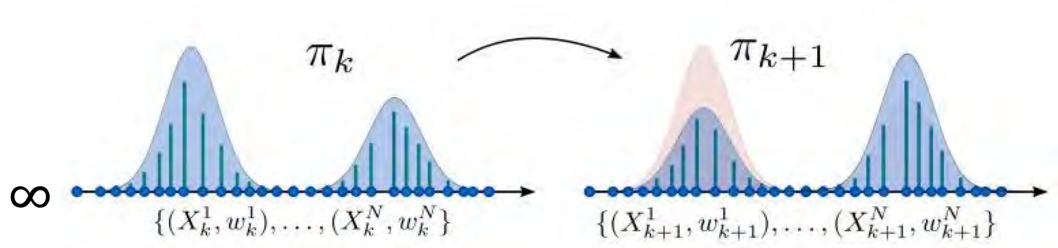


Objective: Compute the conditional probability distribution (posterior) $P(X_t|Y_1, \dots, Y_t)$

Particle Filter

- Approximate π_k with weighted empirical distribution of particles
- Apply the update rule to the particles and weights

Properties:



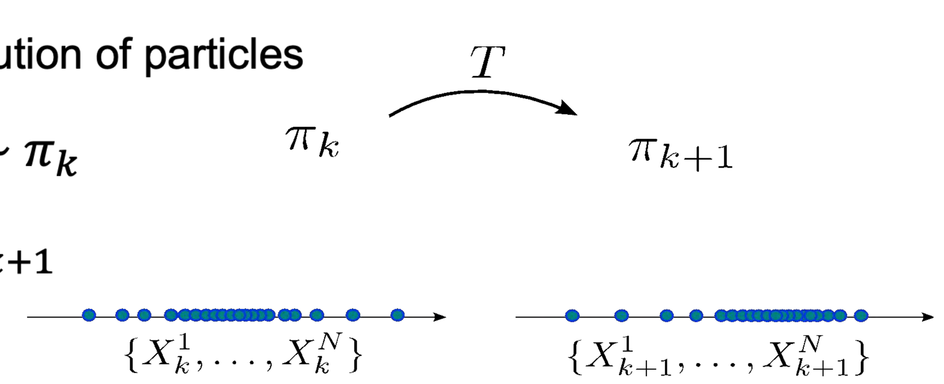
- Exact in the limit as N goes to ∞
- Weight degeneracy (curse of dimensionality)

Optimal Transport Particle Filter

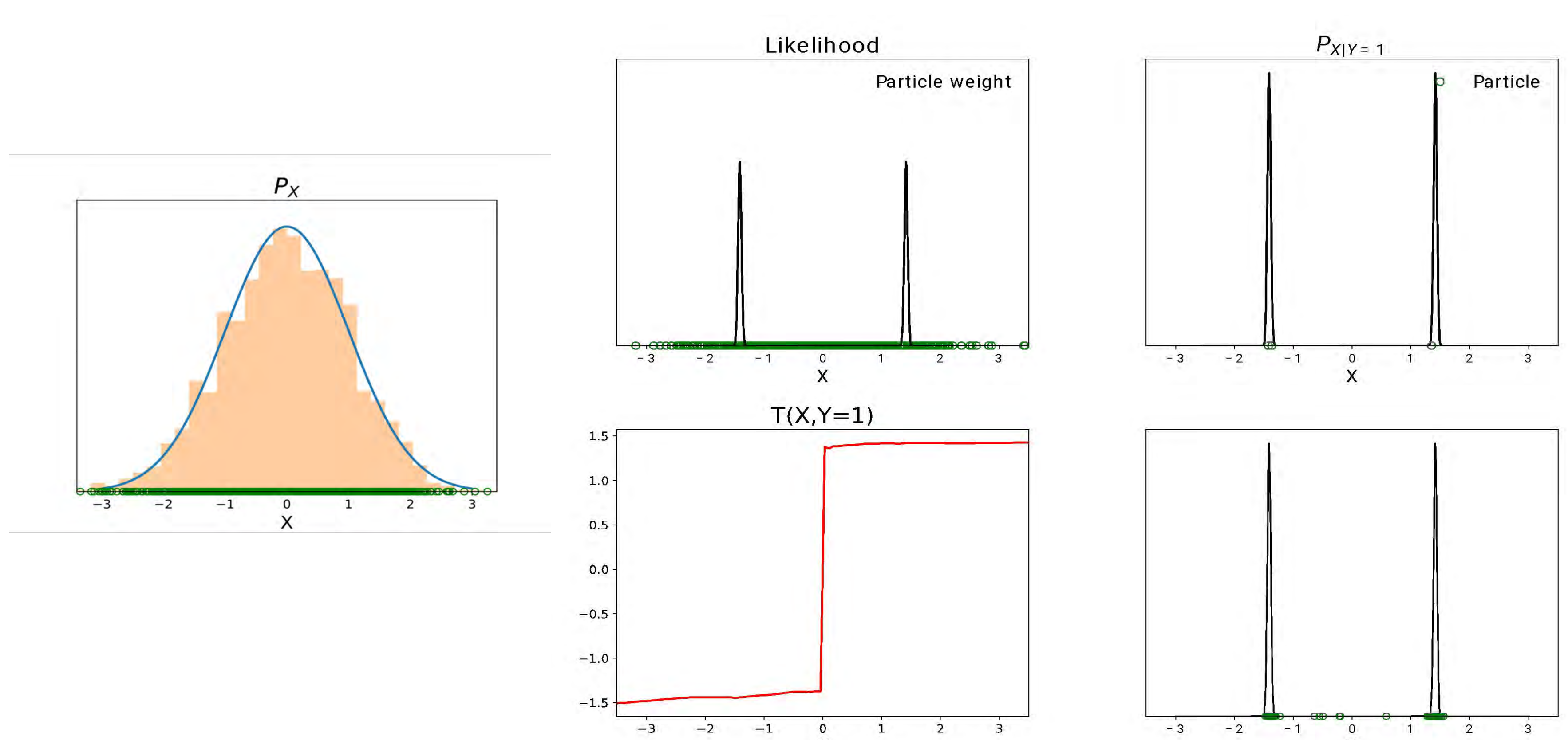
- **Ensemble Kalman filter** avoids curse of dimensionality in linear Gaussian setting
- Can we extend this to **Non-Gaussain** setting?
- Approximate π_k with empirical distribution of particles
- **Main task:** *given*: $\{X_k^1, \dots, X_k^N\} \sim \pi_k$

  generate: $\{X_{k+1}^1, \dots, X_{k+1}^N\} \sim \pi_{k+1}$

  where $\pi_k = P_{X|Y_{1:k}}$



- **OTPF approach:** update particle with the optimal transport map form π_k to π_{k+1}



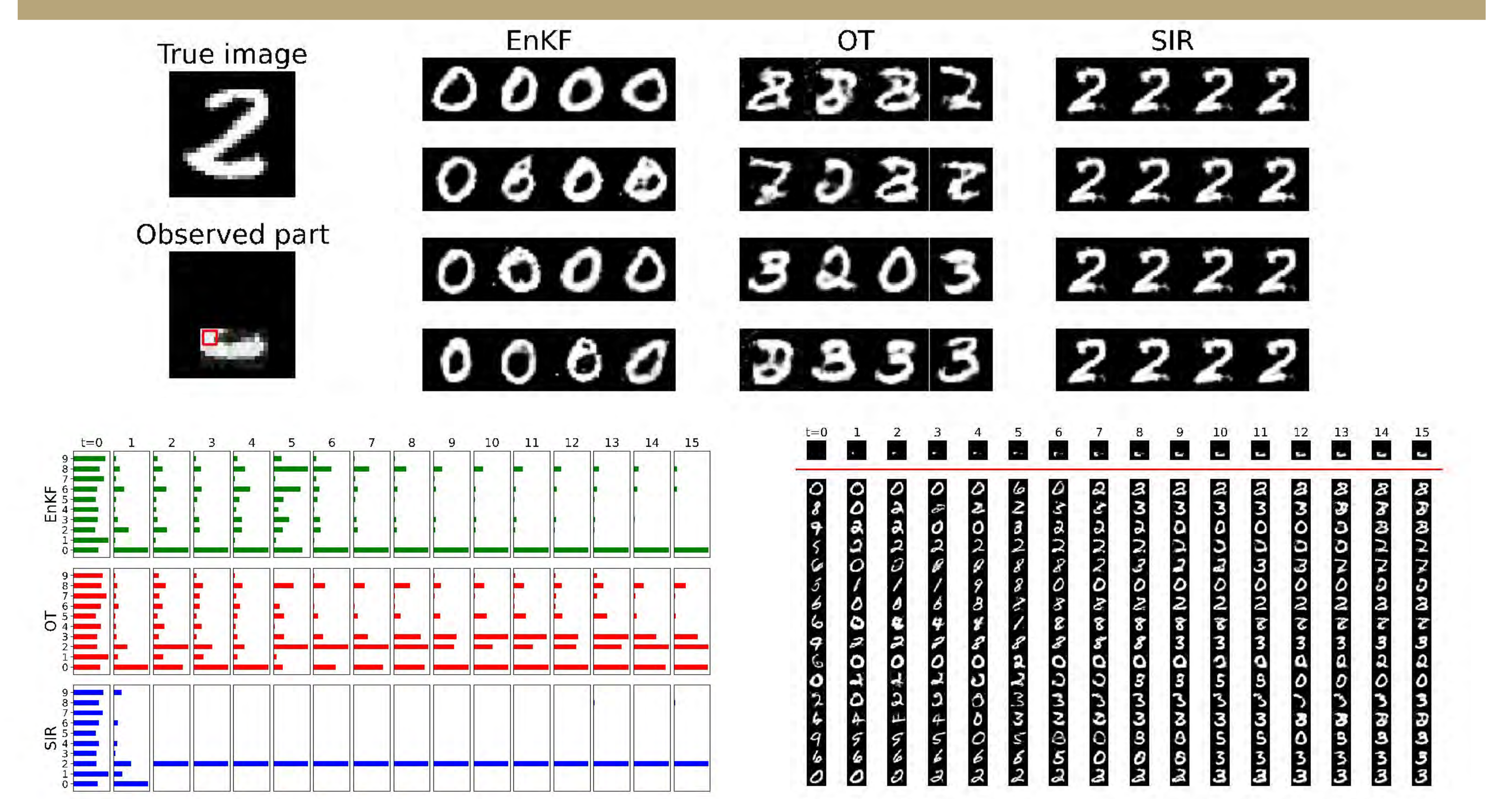
Optimal Transport formulation of the Bayes Law

Bayes Law: $P(X|Y) = \frac{P(X)P(Y|X)}{P(Y)} = \nabla_x \bar{f}(.;Y)_{\#} P_X$

Where $\bar{f} = arg \min_{f \in L^1(X \times Y)} \mathbb{E}_{(X,Y)\sim P_X \otimes P_Y}[f(X;Y)] + \mathbb{E}_{(X,Y)\sim P_{XY}}[f^\star(X;Y)]$

- Only requires samples $(X_i, Y_i) \sim P_{XY}$ (data-driven / simulation based)
- Enable construction of "approximate" posterior distribution
- Allow application of ML tools (Stochastic optimization and Neural Networks)

Numerical Experiment



Future directions of research

- Efficient representations of the transport map
- Test the algorithm on real-world applications
- Develop a distribution feedback control algorithm (rather than pointwise state feedback) that accounts for uncertainty in the system model

References

- **Mohammad Al-Jarrah**, Niyizhen Jin, Bamdad Hosseini, and Amirhossein Taghvaei. "**Nonlinear Filtering with Brenier Optimal Transport Maps**." *Accepted* International Conference on Machine Learning (*ICML*) *2024.*
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