ROBUST TRAJECTORY PLANNING UNDER STATE- AND INPUT-DEPENDENT UNCERTAINTY

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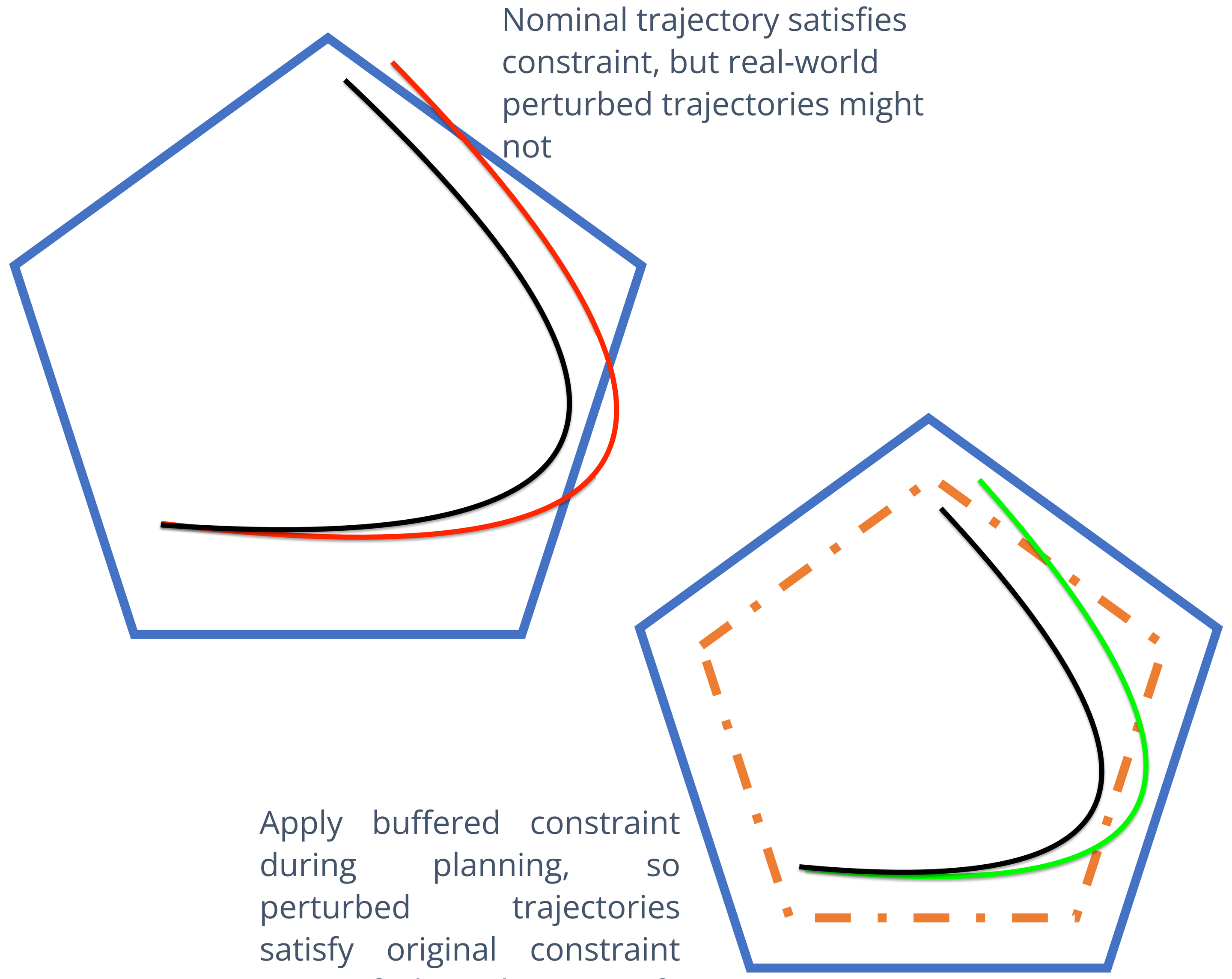
Background and motivation

- Numerical optimization is a powerful technique for constrained trajectory optimization, but the presence of dynamical perturbations can lead to constraint violation if not accounted for
- Convex problems in particular can be solved with fast off-the-shelf solvers, giving strong theoretical results, but these solvers only handle purely deterministic systems
- This work presents a method of reformulating certain robust constrained trajectory optimization problems to exactly equivalent deterministic convex problems

Prior art: constraint buffering



- Much literature exists on methods to compute constraint buffers (including extension to chance constraints), but these methods are often conservative
- Question: can we compute constraint buffers that give exact equivalence to robust satisfaction of original constraint? In other words, how do we find constraint buffers that guarantee feasibility, but only just?

Problem formulation

LTV dynamics: $x_{t+1} = A_t x_t + B_t u_t + E_t(v_t + w_t) + K_t n_t.$

(exogenous input: v_t; perturbations: w_t, n_t)

Perturbation bounds: $0 \preceq n_t \preceq f(x^n_t, u_t)$ (f(x,u) convex and nonnegative)

$G_t w_t \preceq g_t$

We wish to apply a robust linear inequality constraint: $H_t x_t \preceq h_t \quad \forall(\bar w_{t-1}, \bar n_{t-1}) \in \bar{\mathbf{P}}_{t-1}$

Constraint reformulation

$$H_t x_t \preceq h_t \quad \forall(\bar w_{t-1}, \bar n_{t-1}) \in \bar{\mathbf{P}}_{t-1}$$

is equivalent to:

$$\max_{(\bar w_{t-1}, \bar n_{t-1}) \in \bar{\mathbf{P}}_{t-1}} e_i^T H_t x_t \leq e_i^T h_t, \quad i = 1, \ldots, m_t$$

which (bounding above through duality) is equivalent to:

$$\begin{aligned} Z_t \bar G_{t-1} &= H_t \bar E_{t-1} \\ \Lambda_t &\succeq H_t \bar K_{t-1} \\ \underbrace{Z_t \bar g_{t-1} + \Lambda_t \bar f(\bar x^n_{t-1}, \bar u_{t-1})}_{\text{uncertainty buffer}} &\preceq \underbrace{h_t - H_t x^n_t}_{\text{nominal state constraint}}. \end{aligned}$$

which (minimizing lambda in closed form) is equivalent to:

$$\begin{aligned} Z_t &\succeq 0, \\ Z_t \bar G_{t-1} &= H_t \bar E_{t-1}, \\ Z_t \bar g_{t-1} + \Gamma_t \bar f(\bar x^n_{t-1}, \bar u_{t-1}) &\preceq h_t - H_t x^n_t \end{aligned}$$

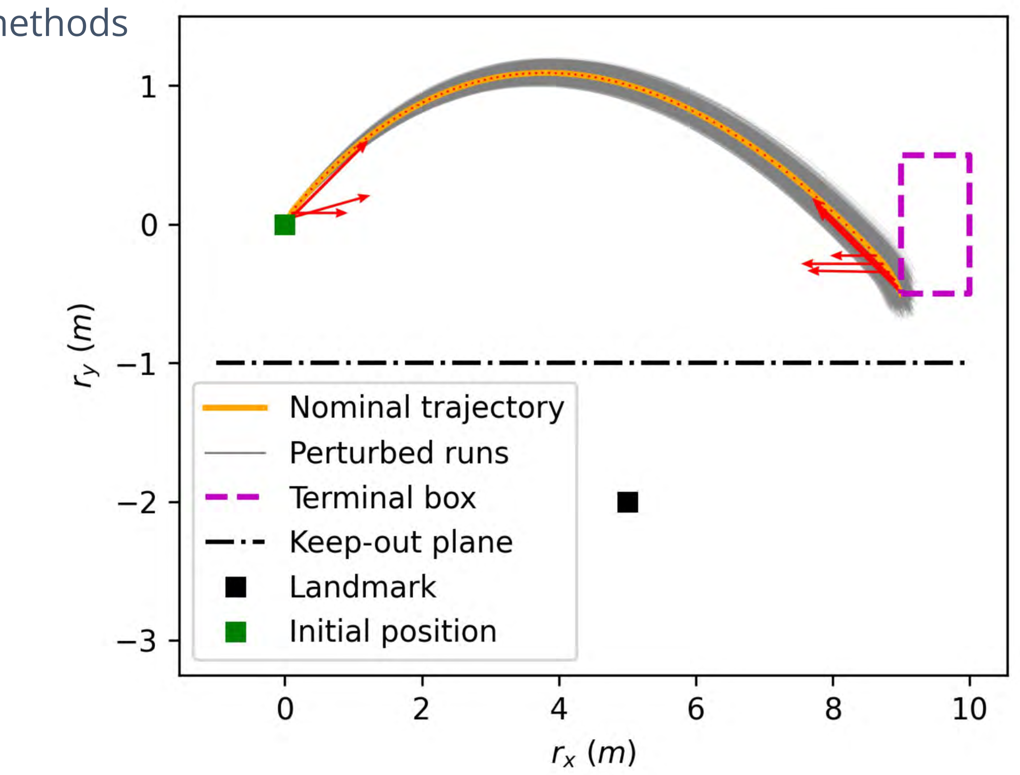
where

$$\Gamma_t = \max\left(0, H_t \bar K_{t-1}\right).$$

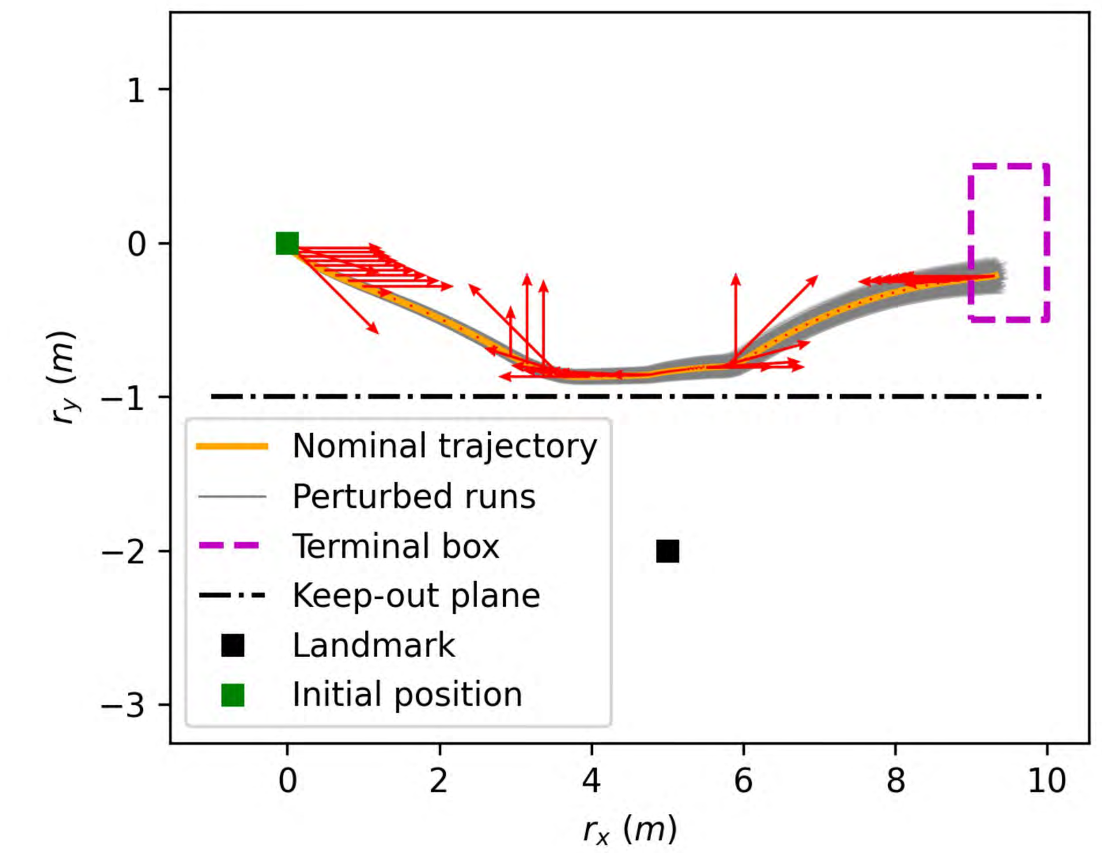
Convex!

Numerical results

- Dynamics: linearized and discretized Clohessy-Wiltshire dynamics (which model the relative dynamics of one spacecraft about another in a circular orbit)
- Constraints: initial state, final bounding box, keep-out plane to separate trajectory from "landmark" (see next point)
- Perturbation bounds: increase with increasing distance from a landmark point; roughly models effect of state uncertainty under relative navigation methods



Trajectory planned without accounting for perturbations; many Monte Carlo runs violate the final bounding box constraint.



Trajectory planned accounting for perturbations; all Monte Carlo runs respect all constraints.

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